TEMPERATURE FIELDS IN A PIPE WALL IN THE CASE OF RADIATIVE HEATING AND A SUPERCRITICAL WORKING-MEDIUM PRESSURE

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The problem on determination of the temperature field in the tube wall with nonuniform heat flux along the perimeter at the external surface and heat transfer to the medium of supercritical pressure at the internal surface is solved. The heat transfer coefficient to the supercritical pressure medium depends on the heat flux at the internal tube surface, process and design parameters. The results of calculations are compared with experimental data and calculations by other methods.

Heat exchange with a medium at supercritical pressure in a radiatively heated pipe has certain distinctive features: first, the heat-transfer coefficient at high specific heats, i.e., for flow enthalpies $250 \leq i_f \leq 650$ kcal/kg, depends on the heat flux, the average-mass velocity, the flow enthalpy, the pressure, and structural parameters; second, radiatively heated pipes may not be heated uniformly over the perimeter of the pipe. These two facts significantly complicate a determination of the temperature fields in the pipe wall.

Under these conditions, a calculation of the temperature fields in the wall by the procedure of [1] neglects the nonuniform distribution of the heat-transfer coefficient α along the pipe perimeter; i.e., the coefficient α is assumed constant and equal to its value at the frontal line (the line at the pipe wall which is parallel to the pipe axis and which lies at the front of the pipe.

Furthermore, the values of α determined by the method of [1] are frequently lower than the actual values.

The temperature fields in the pipe wall can be determined more rigorously by using local values of the heat-transfer coefficient along the perimeter and by using the more accurate method of [2, 3] for determining α.

The dependence of α on the heat flux to the inner surface and other parameters of the structure and the process was found for uniformly heated pipes in [2, 3]. The experimental data of [4, 5] show that, for equal local heat fluxes at the inner wall, the heat-transfer coefficients at the frontal line are not the same for the cases of uniform and nonuniform heating over the range of volume-averaged flow enthalpies $i_f = 350-550$ kcal/kg. Under these conditions the heat-transfer coefficients are higher in the case of nonuniform heating than in the case of uniform heating. If the heat-transfer coefficient obtained for uniform heating is used in the boundary condition in a determination of the temperature fields in a nonuniformly heated pipe wall, the calculated temperature at the frontal wall will be slightly higher than the actual temperature over the range $i_f = 350-550$ kcal/kg.

Turning immediately to the solution of the problem, we note that the steady-state temperature field in the pipe wall is described by

$$\frac{1}{r}\frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \psi^2} = 0 \qquad (*)$$

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with the boundary conditions

$$\left.\frac{\partial t}{\partial \psi}\right|_{\psi=0,\pi} = 0, \tag{1}$$

$$\lambda \left.\frac{\partial t}{\partial r}\right|_{r=r_{\text{in}}} = \alpha(\psi)\cdot t, \tag{2}$$

$$\lambda \left.\frac{\partial t}{\partial r}\right|_{r=r_{\text{o}}} = q(\psi). \tag{3}$$

We see from Eqs. (1)-(3) that α and q are even functions. If we assume that α and q can be expanded in convergent Fourier series, we would have

$$\alpha(\psi) = \frac{\alpha_0}{2} + \sum_{n=1}^{\infty} \alpha_n \cos n\psi,$$

$$q(\psi) = \frac{q_0}{2} + \sum_{n=1}^{\infty} q_n \cos n\psi,$$

where

$$\alpha_n = \frac{2}{\pi}\int_0^{\pi} \alpha(\psi)\cos n\psi d\psi,$$

$$q_n = \frac{2}{\pi}\int_0^{\pi} q(\psi)\cos n\psi d\psi.$$

We introduce the quantities $\alpha_{-n} = \alpha_n$. Using (1), we can write the formal solution of Eq. (*) as

$$t(r, \psi) = \sum_{n=1}^{\infty} (A_n r^n + B_n r^{-n})\cos n\psi + A_0 \ln r + B_0.$$

Transforming the original equations and boundary conditions, we find that the coefficients A_i and B_i must satisfy

$$A_0 = \frac{r_1 q_0}{2\lambda},$$

$$A_n r_1^{n-1} - B_n r_1^{-n-1} = \frac{q_n}{n\lambda},$$

$$A_0 \ln r_2 + B_0 = \frac{r_1}{r_2}\frac{q_0}{\alpha_0} - \sum_{k=1}^{\infty}\frac{\alpha_k}{\alpha_0}(A_k r_2^k + B_k r_2^{-k}),$$

$$\lambda n(A_n r_2^{n-1} - B_n r_2^{-n-1}) = \frac{1}{2}\sum_{k=1}^{\infty}(A_k r_2^k + B_k r_2^{-k}) \times$$

$$\times\left(\alpha_{k+n} + \alpha_{k-n} - \frac{2\alpha_k\alpha_n}{\alpha_0}\right) + \frac{r_1}{r_2}\frac{q_0}{\alpha_0}\alpha_n,$$

where n = 1, 2, ...

We introduce the new quantities

$$y_n = \lambda n(A_n r_2^{n-1} - B_n r_2^{-n-1}), \quad n = 1, 2, \ldots$$

Then the solution of the problem reduces to the solution of the infinite system of equations

$$y_n = b_n - \sum_{k=1}^{\infty} C_{kn} y_k, \quad k, n = 1, 2, \ldots,$$

where

$$b_n = \frac{q_0\alpha_n}{\alpha_0 a} + \frac{r_1}{\lambda}\sum_{k=1}^{\infty}\frac{q_k}{k}\frac{a^k}{1-a^{2k}}\left(\alpha_{k+n} + \alpha_{k-n} - \frac{2\alpha_k\alpha_n}{\alpha_0}\right),$$

$$C_{kn} = \frac{r_2}{2\lambda k}\frac{1+a^{2k}}{1-a^{2k}}\left(\alpha_{k+n} + \alpha_{k-n} - \frac{2\alpha_k\alpha_n}{\alpha_0}\right),$$

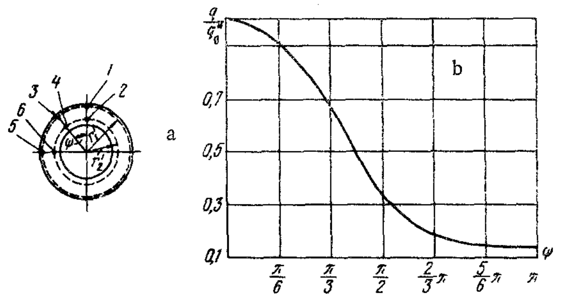

Fig. 1. a) Cross section of the pipe; b) relative distribution of heat load over the perimeter of the outer pipe surface.

$$a = \frac{r_2}{r_1}.$$

After calculating y_n, we find the solution of the problem from

$$t(r, \psi) = \sum_{k=1}^{\infty} C_k(r) \cos k\psi + C_0(r),$$

where

$$C_0(r) = \left(\frac{q_0}{\alpha_0 a} - \sum_{k=1}^{\infty} \frac{\alpha_k}{\alpha_0} C_k(r_2)\right) + \frac{r_1 q_0}{2\lambda} \ln\left(\frac{r}{r_2}\right),$$

$$C_k(r) = A_k r^k + B_k r^{-k} = \frac{1}{k\lambda(1 - a^{2k})}\left[(q_k r_1 - y_k r_2 a^k)\left(\frac{r}{r_1}\right)^k + (q_k r_1 a^k - y_k r_2)\left(\frac{r_2}{r}\right)^k\right].$$

Since $q(\psi)$ and $\alpha(\psi)$ can be tabulated, we must calculate the Fourier coefficients for these functions; to do this we use the quadrature equation with equidistant nodes, based on the algebraic interpolation of [6]:

$$\int_0^{\pi} f(x) \cos kx dx = \frac{1 - \cos kh}{hk^2}\left[f_0 + 2\sum_{i=1}^{n-1} f_i \cos(k, ih) + (-1)^n f_n\right],$$

where $h = \pi/n$.

The accuracy of this equation is governed primarily by the accuracy with which the graph of the function over the integration interval is approximated by a broken line with slope changes at the points given in the table. If the functions α and q are sufficiently smooth, convergence of the series is ensured, and the system has a solution, as has been verified in numerical computer "experiments."

In the course of the calculations, a check was made to see that the boundary conditions were satisfied for the resulting solution. Use of the tabulated Fourier coefficients in the calculations smooths the errors in the specification of these functions. An accuracy sufficient for engineering purposes is achieved with ~10 terms of the expansion.

The function $q(\psi)$ can be determined by the procedure of [7]. The function $\alpha(\psi)$ can be written as follows, according to an analysis of numerical calculations on the basis of [3]:

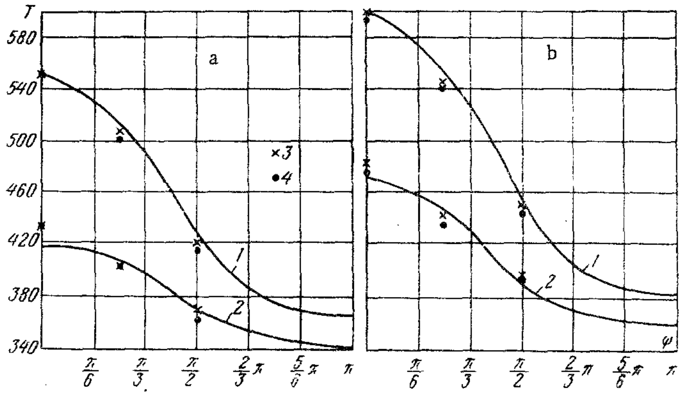

Fig. 2. Comparison of calculated and experimental temperatures. The curves are calculated (curves 1 for radius r_1' and curves 2 for r_2') and the points are experimental. The values of ρW [kg/(m$^2\cdot$sec)], P (kg/cm^2), i_f (kcal/kg), and q_0^* [kcal/(m$^2\cdot$h)], respectively, are: a: curves) 2000, 260, 340, $0.6\cdot10^6$; 3) 2010, 260, 340, $0.61\cdot10^6$; 4) 2000, 260, 343, $0.595\cdot10^6$. b: curves) 1000, 260, 350, $0.58\cdot10^6$; 3) 1000, 259, 353, $0.58\cdot10^6$; 4) 940, 259, 350, $0.57\cdot10^6$.

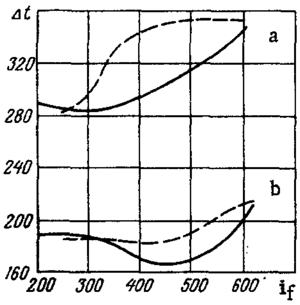

Fig. 3. Difference between the temperature of the outer front point of the pipe and the flow temperature, $\Delta t = T_w - T_f$, as a function of the enthalpy of the medium [P = 260 kg/cm^2, $\rho W = 1000$ kg/m^2·sec.] a) $q_0^* = 0.4 \cdot 10^6$ kcal/(m^2·h); b) $q_0^* = 0.6 \cdot 10^6$ kcal/(m^2·h). Solid curves) Calculated by the procedure of the present paper; dashed) by the procedure of [1].

$$\alpha = 7965 \left(\frac{\rho W}{1000}\right)^{0,8} \left(\frac{0,02}{d_{in}}\right)^{0,2} \left[1 - 0,15 \exp\left(-\frac{2,32}{B}\right)\right] \frac{\alpha}{\alpha_{200}},$$

where

$$\frac{\alpha}{\alpha_{200}} = f(B, i_f, P) \quad \text{is tabulated, and}$$

$$B = \frac{q_{in}(\psi) \cdot 10^{-3} d_{in}^{0,3}}{(\rho W)^{0,8+0,03(\rho W / q_{in}(\psi) \cdot 10^{-3})^{1,2}}}.$$

If the flow enthalpy, the pressure, the average-mass velocity, and the inner pipe diameter are given, the quantity $\alpha = f[q_{in}(\psi)]$ is a known function of the heat flux at the inner surface; in turn, we have $q_{in}(\psi) t(r_2, \psi)$. Accordingly, α is specified implicitly and is determined from the equation as a function of the temperature distribution on the inner pipe wall. Therefore, the problem is solved by the method of successive approximations.

For the first approximation we adopt $q_{in}^0(\psi) = \text{const}$, where q_{in}^0 can be chosen comparatively freely, e.g., $q_{in}^0 = q_0^*(d_o/d_{in})$. On the basis of this initially specified value of q_{in}^0 we determine $\alpha^{(1)} = f(q_{in}^0)$, solve the system of equations above, and find the temperature field in the wall, $t^{(1)}(r, \psi)$.

Then we calculate

$$\alpha^{(2)} = f(q_{in}^{(1)}),$$

where

$$q_{in}^{(1)} = \alpha^{(1)} t^{(1)}(r_2, \psi),$$

$$\alpha^{(i+1)} = f(q_{in}^{(i)}),$$

where

$$q_{in}^{(i)} = \alpha^{(i)} t^{(i)}(r_2, \psi).$$

The calculation is stopped when $\max_{\psi} |t^{(i)}(r, \psi) - t^{(i-1)}(r, \psi)| < \varepsilon$; we finally have

$$T(r, \psi) = T_f + t(r, \psi).$$

In the range of coolant parameters in which there is an abrupt degradation of heat transfer at the frontal line, calculations on the basis of these equations may turn out to be unstable, with a finite but undamped oscillation of the heat flux. In this case it is necessary to introduce the relaxation time τ and to calculate

$$q^{(i)} = q^{(i-1)} + \tau [\alpha^{(i)} t^i (r_2, \psi) - q^{(i-1)}].$$

Convergence of the process can be hastened by using a varying relaxation coefficient, choosing it to vary in the optimum manner on the basis of experiment.

Calculations carried out by this procedure were compared with experimental data, obtained by I. E. Semenovker on a PK-41 boiler. The measurements were made on a pipe with $d_o/d_{in} = 36/20$ (Fig. 1). The pipe temperature was measured at radii $r_2' = 17.5$ mm and $r_1' = 21.5$ mm in three radial directions at intervals of 45°. Points 1 and 2 lie on the direction of maximum heat flux. The relative distribution of the heat flux over the outer surface of the pipe is shown in Fig. 1b.

Comparison of the calculated temperatures of the pipe metal at r_1' and r_2' with the experimental points (Figs. 2a and 2b) reveals a satisfactory agreement. Figure 3 compares the temperatures in the outer frontal point of the pipe as determined by the present procedure and by the procedure of [1].

NOTATION

t	is the difference between the temperature at a given point of the pipe and the flow temperature;
T_f	is the flow temperature;

T	is the temperature at the given point of the pipe;
i_f	is the flow enthalpy;
P	is the pressure;
q_0^*	is the heat flux at the frontal point at the outer surface;
q_{in}	is the heat flux at the inner surface of the pipe;
q	is the heat flux at the outer surface;
α	is the heat-transfer coefficient;
λ	is the thermal conductivity of pipe material;
ρW	is the average-mass flow velocity;
$r_o = r_1$	is the outer radius of pipe;
$r_{in} = r_2$	is the inner radius;
d_o and d_{in}	are the outer and inner pipe diameters, respectively;
ψ	is the angle measured from the frontal point of the tube.

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